NOTATION

B = $\rho gh/\gamma$, dimensionless film thickness
g = gravitational acceleration, m/s^2
h = film thickness, m
N_{Re} = $h <v> \rho/\mu$, film Reynolds number
N_{Th} = $(\gamma^3/\rho g^2\mu^2)^{1/3}$, Thomson number
q = volumetric flow/unit width, m^2/s
Q = $q\, \rho^2 g^2 \mu/\gamma^3$, dimensionless flow rate
R = $h(g\rho^2/\mu^2)^{1/3}$
U = $v\, \mu/\gamma h$, dimensionless fluid velocity
v = fluid velocity, m/s
$<v>$ = q/h = mean fluid velocity at a cross-section, m/s
v_h = velocity of a thickness locus, m/s
V = $<v>\mu/\gamma h = q\mu/\gamma h^2$ = dimensionless mean velocity
W = $v_h\, \rho g\mu/\gamma^2$, dimensionless locus velocity
y = distance from solid surface, m
Y = y/h = dimensionless distance from surface

Greek Letters

γ = surface tension gradient, N/m^2
μ = dynamic viscosity, Pa s
ρ = density, Kg/m^3

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A Simple Expression for the Velocity Distribution in Turbulent Flow in Smooth Pipes

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The velocity distribution in turbulent flow in smooth pipes is of great interest because it provides a test of the attainment of fully developed flow, necessary information for the detailed calculation of heat transfer, component transfer, and chemical conversions in turbulent flow, and the basis for the development of the highly successful analogies between momentum transfer and heat and component transfer.

For the region adjacent to the wall the assumption of laminar motion and a negligible variation in shear stress leads to

$$u^+ = y^+ \tag{1}$$

Prandtl (1933) derived the expression

$$u^+ = A + B \ln y^+ \tag{2}$$

by assuming that the turbulent shear stress was proportional to $(ydu/dy)^2$ and that the viscous shear stress and the radial variation in the total shear stress were negligible. Despite these extreme assumptions, Equation (2) represents the form of the velocity distribution far outside the laminar boundary reasonably well.

Equations (1) and (2) can be patched together to yield a complete representation by the proper choice of A and B. A considerably better representation is obtained by using Equation (1) out to $y^+ = 5$, Equation (2) with one set of coefficients from 5 to 30 and with a second set

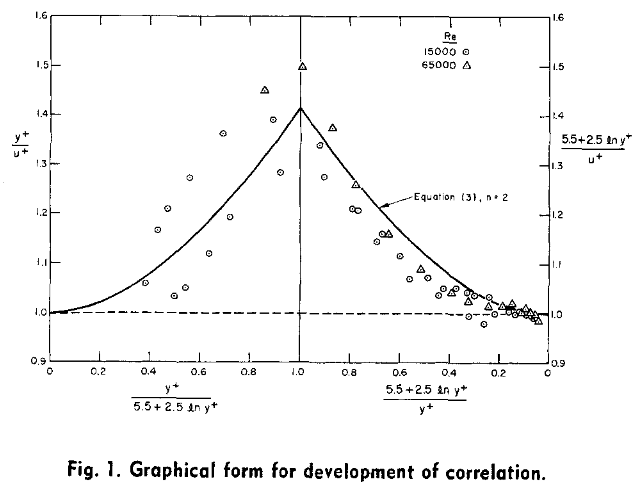

Fig. 1. Graphical form for development of correlation.

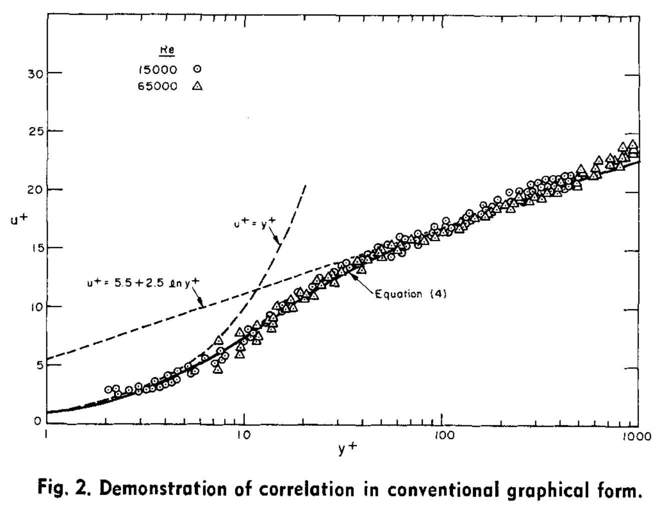

Fig. 2. Demonstration of correlation in conventional graphical form.

of coefficients for $y^+ > 30$. However such segmental representations produce discontinuities in du/dy at the intersections which in turn result in very awkward expressions for heat and component transfer.

Many functions with continuous derivatives have been proposed to represent the velocity distribution from the wall to the centerline. Most of these suffer from inaccuracy or complexity.

The general expression proposed by Churchill and Usagi (1972) for correlation of rate data can be used to construct an empirical equation which interpolates continuously between Equations (1) and (2). The trial expression is

$$\frac{y^+}{u^+} = \left[1 + \left(\frac{y^+}{A + B \ln y^+}\right)^n\right]^{1/n} \tag{3}$$

Representative experimental data from the work of Abbrecht and Churchill (1960) are plotted in the suggested form in Figure 1 with $A = 5.5$ and $B = 2.5$. A value of $n = 2$ is seen to provide a reasonable overall fit yielding a correlation which can be rearranged as

$$\frac{1}{u^{+2}} = \frac{1}{y^{+2}} + \frac{0.16}{\ln^2(9y^+)} \tag{4}$$

The coordinates of Figure 1 exaggerate the scatter of the data and the deviations from Equation (4). The more conventional coordinates of Figure 2 confirm that a very good representation is provided by Equation (4).

Equation (4) is in itself somewhat unwieldy in form but: (1) it is continuous and has continuous derivatives for $y^+ > 0.11$; (2) it approaches the two well-known limiting solutions asymptotically; (3) it does not require integration or the evaluation of complex functions and (4) it is convenient for slide rule or machine calculations. The singularity at $y^+ \cong 0.11$ is not of practical concern in calculations since Equation (1) can be used rather than Equation (4) for $y^+ < 5$.

NOTATION

A = empirical coefficient [Equation (2)], dimensionless
B = empirical coefficient [Equation (2)], dimensionless
n = empirical exponent [Equation (3)], dimensionless
u = velocity, m/s
u^+ = $u/\sqrt{\tau_w/\rho}$, dimensionless
y = distance from wall, m
y^+ = $y\sqrt{\tau_w/\rho}/\nu$
ν = kinematic viscosity, m²/s
ρ = density, kg/m³
τ_w = shear stress on wall, N/m²

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Theoretical Non-Newtonian Pipe-Flow Heat Transfer

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Numerous investigations of turbulent non-Newtonian pipe-flow heat transfer have resulted in correlations of heat transfer to purely viscous fluids based on empirically fitting turbulent heating data or on the analogy between energy and momentum transport. Correlations of momentum transport are then used in the latter case to provide predictions of heat transfer. The present analysis is based on integration of the momentum and energy equations across the pipe diameter to yield a general solution to the enthalpy profiles and heat transfer for a power-law fluid under constant wall flux conditions. The velocity profiles are calculated using the author's formulation for turbulent momentum exchange. The objective of the study is to illustrate the effectiveness of a theoretical model in predicting non-Newtonian heat transfer in a way analogous with the theoretical treatment of Newtonian fluid flow.

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Metzner and Friend (1959) provided the first semitheoretical analysis of the problem based on the work of Dodge and Metzner and their previous work on turbulent pipe flow of Newtonian fluids (Friend and Metzner, 1958). The approach, based on analogy between heat and momentum exchange, permitted extension of the treatment of non-Newtonian fluids beyond the limited ranges dealt with previously on a purely empirical basis.

Clapp investigated the problem experimentally and developed semi-empirical temperature profiles as a function of pipe radius. In doing so, he formulated the local shear stress as the sum of the power law non-Newtonian shear and the purely turbulent shear based on Prandtl's momentum transfer theory. This is in analogy with Prandtl's approach for a Newtonian fluid. Wells (1968) extended the analogy between energy and momentum transport to include drag reducing fluids using a correlation of the viscous sublayer thickness and friction factor for dilute solutions of polymers (Meyer, 1966). Wells makes it clear that any treatment of the energy transfer of general non-